M.Phil. Mathematics

under

Choice Based Credit System

2018 - 19

Syllabus

Ramanujan Institute for Advanced Study in Mathematics

University of Madras

June 2018

M.Phil. Mathematics(CBCS) 2018 - 2019

Course Code	Course Title	Core	Credit			
			L	T	P	C
MSI C031	Algebra	C	4	1	0	5
MSI C032	Analysis	C	4	1	0	5
MSI C033	Topology and Geometry	C	4	1	0	5
MSI C034	Dissertation and Viva Voce	C				21

MSI C031	ALGEBRA	4	1	0	5

Pre-requisite :

Group actions, free groups, fundamental theorem of finitely generated abelian groups, Euclidean domain, principal ideal domains unique factorization domain, polynomial rings, irredicibility of polynomials over a field, division algorithm.

Unit I

Matrix rings , Group rings, rings of fractions, Chinese remainder theorem Modules, Quotient modules, Module homomorphism, generation of modules, direct sums and free modules.

Unit II

Tensor product of modules, exact sequences, projective modules, injective modules, flat modules, tensor algebra, symmetric algebra, exterior algebra.

Unit III

Noetherian rings and affine algebraic sets, radicals and affine varieties, integral extensions and Hilbert's Nullstellensatz, localisation, prime spectrum of a ring.

Unit IV

Artinian rings discrete valuation rings, Dedekind domains, homology cohomology of groups, Ext, Tor.

Unit V

Linear actions and modules over group rings, Wedderburn's theorem and consequences, character theory and orthogonality relations.

References:

1. Abstract Algebra, David Dummit & Richard Foote, Wiley Student edition.
2. Commutative algebra, M. Atiyah, I. Macdonald, Hachette UK 1994.
3. Commutative algebra, with a view toward algebraic geometry, D. Eisenbud.

MSI C032	ANALYSIS	4	1	0	5

Unit I : Abstract Integration:
Sigma algebra, Measure, Elementary properties of measures, Integration of positive functions, Integration of Complex functions, Monotone convergence theorem, Fatou's lemma, Dominated convergence theorem, Role played by sets of measure zero.

Unit II : Positive Borel Measures and Lebesgue Spaces:
Review of topological concepts (Compactness, Hausdorff criteria etc.), Uryshon's Lemma, Partition of unity, Riesz representation theorem, Regularity of Borel measures, Lebesgue measure on $\mathbb{R}^n$ Continuity property of Borel measurable functions (Luzin's theorem, Vitali Caretheodory theorem), Concept of convexity and Jensen's inequality, The Lp spaces, Approximation by continuous functions.

Unit III : Complex Measures and Differentiation:
Complex measure, Total variations, Positive and negative variations, Absolute continuity, Radon-Nikodym theorem, Bounded linear functional on L_p spaces, Riesz representation theorem. Differentiation of measures, Lebesgue points, Absolute continuity, Fundamental theorem of Calculus, Differentiable transformations.

Unit IV: Integration on Product Space and Fourier Transforms:
Product sigma algebra, product measures, The Fubini's theorem, Completion of product measures,Convolution, Distribution functions, Definition and properties of Fourier transformation, Inversion theorem, Plancherel theorem , The Banach Algebra L_1, Complex homomorphisms.

References:

1. W. Rudin, Real and Complex Analysis, Third Edition, Mc GrawHill Book company 1987.
2. E. Hewitt, K.R. Stromberg, Real and Abstract Analysis, Springer - Verlag, 1975.
3. G.B. Folland, Real Analysis, John Wiley & Sons, 1984.

MSI C033	TOPOLOGY AND GEOMETRY	4	1	0	5

Unit I

Contractible spaces, homotopy of paths, fundamental group, induced homomorphisms, covering spaces, path lifting lemma, lifting correspondence, fundamental group of the circle.

Unit II

Simplicial Complexes, subdivision, barycentric subdivision, simplicial approximation theorem, fundamental group of a simplical complex.

Unit III

Differentiable manifold definition & Examples, Differentiable maps, Tangent spaces and differentials, Tangent vectors, The differential of a map, Orientability and tangent spaces, immersions, submersions and submanifolds.

Unit IV

Survey of multilinear algebra, vector bundles on manifolds, vector fields on manifolds, push- forwards of vector fields, the Lie Bracket, Differential forms.

Unit V

The Exterior differential, Pullbacks, Integration on manifolds, partitions of unity, integrating differential forms, conservastive vector fields, stoke's theorem.

References:

1. Topology, Munkres, J., Pearson
2. Lecture Notes in elementary Topology and Geometry, singer and Thorpe, Springer.
3. Differential geometry of manifolds, Stephen Lovett, Taylor & Francis, 2010.